The Rise of Intangible Capital and the Macroeconomic Implications

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Online Appendix

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I.I Data

I.I.I Main Sample, Variables, and Summary Statistics

We use the Compustat dataset from 1980 to 2015. We linearly interpolate `SALE`, `COGS`, `XSGA`, `EMP`, `PPEGT`, `PPENT`, `XRD`, `INTAN`, `GDWL`, and `AM`. We exclude utilities (SIC codes between 4900 and 4999) because their prices are heavily regulated. We also exclude financial firms (SIC codes between 6000 and 6999) because their balance sheets are dramatically different from those of other firms. For data quality, we interpret as mistakes if `SALE`, `PPEGT`, `PPENT`, `COGS`, `EMP`, or `XSGA` are zero, negative, or missing, and we drop those observations. Moreover, if `XSGA` is missing or negative, we drop it as well. Finally, if `XRD`, `INTAN`, `AM`, or `GDWL` are negative or missing, we treat them as zeros. To obtain a real measure of the main variables, we deflate them with the GDP deflator, and we deflate investment in tangible and intangible capital by the appropriate deflators.[1] Table I presents a few basic summary statistics for a few leading variables used in our analysis.

Table I: Summary Statistics (1980-2015)

	Sales	Cost of Goods Sold	Employment	Tangible Capital Stock	Intangible Capital Stock
Mean	2,310,810	1,572,800	7,966	1,572,164	284,519
25$^{\text{th}}$ Percentile	27,495	14,880	131	8,004	2
Median	153,005	89,241	686	51,066	3,098
75$^{\text{th}}$ Percentile	809,728	510,199	3,625	349,551	34,060
No. Obs.	188,151	188,151	188,151	188,151	188,151

Note. Summary statistics of cleaned Compustat dataset between 1980 and 2015. All variables are in thousands of US$. Sales and Cost of Goods Sold are deflated with the GDP deflator with base year 2012, and both types of capital stock are deflated using the appropriate investment deflator with base year 2012.

I.I.II User Cost of Tangible and Intangible Capital

One of the challenges of using the cost shares approach to estimate the firm-level production function is that it requires a measure of the user cost of capital. To this end, we define the user cost of capital as

$$r_{j,t} = i_t - \mathbb{E}_t \pi_{t+1} + \delta_j, \quad j \in \{T, I\}, \tag{1}$$

where i_t equals the nominal interest rate, $\mathbb{E}_t \pi_{t+1}$ is expected inflation at time t, and δ_j is the capital-specific depreciation rate. We take the annual Moody's Seasoned Aaa Corporate Bond Yield as an empirical proxy of the nominal interest rate and use the annual growth rate of the Investment Non-residential Price Deflator to calculate expected inflation.[2,3] The depreciation rate of tangible capital is calibrated to $\delta = 0.07$, and the firm-level depreciation rate of intangible capital is computed as a

[1] Deflators are taken from the NIPA tables.

[2] Moody's Seasoned Aaa Corporate Bond Yield: https://fred.stlouisfed.org/series/AAA. The Investment Price Deflator: https://fred.stlouisfed.org/series/A008RD3Q086SBEA.

[3] We estimate an AR(1) process on the annual growth rate of the Investment Nonresidential Price deflator and define the contemporaneous expected inflation as $\mathbb{E}_t \pi_{t+1} = \mu + \rho \pi_t$.

weighted average of the depreciation rates used to construct the intangible capital stock.[4]

I.I.III Intangible Capital Measurement and Accounting Standards

Measuring intangible capital is a difficult task as the accounting standards (US GAAP) are insufficient to satisfactorily book the intangible assets on the balance sheets. It is well established in the corporate finance literature that intangible assets are not fully captured on firms' balance sheets because of the anachronism of the US GAAP. [5] In this section of the appendix, we explain in detail which assumptions are needed to compute intangible capital at the firm level using the balance sheet for stocks and the income statements for flows.

To introduce our main measure, we have to clarify that intangible capital is intrinsically different from tangible capital as a significant part of it is internally generated by the firms. For nearly all internally generated intangible assets, such as knowledge and organizational capital, accounting standards differ significantly from tangible assets. All purchases of tangible assets are recorded on the balance sheet at their purchased price and depreciated over their useful life. Conversely, internal intangible capital investments, such as firms' R&D expense, advertising, or training of employees, are fully expensed in the period they are incurred.[6]

Figure I: Advertising Expenses of Coca Cola

Selling, General and Administrative Expenses

The following table sets forth the significant components of selling, general and administrative expenses (in millions):

Year Ended December 31,		2016		2015		2014
Stock-based compensation expense	$	**258**	$	236	$	209
Advertising expenses		**4,004**		3,976		3,499
Selling and distribution expenses		**5,177**		6,025		6,412
Other operating expenses		**5,823**		6,190		7,098
Selling, general and administrative expenses	$	**15,262**	$	16,427	$	17,218

For instance, the Coca-Cola Company spends several billion dollars each year to maintain and promote its products and brands, such as Coca-Cola and Dasani. These are the assets of the firm that will generate future benefits in the form of higher margins and increased sales volume. However, the Coca-Cola Company is not allowed to recognize these assets on its balance sheet. Figure I shows that Coca-Cola spent around $4 billion in advertising in 2016. We also provide the example of Google Inc.,

[4]The firm-level depreciation rate of intangible capital is computed as

$$\delta_{I,ft} = \frac{k_{ft}^{R\&D}}{k_{ft}^{R\&D} + k_{ft}^{BS}} \delta_s^{R\&D} + \frac{k_{ft}^{R\&D}}{k_{ft}^{R\&D} + k_{ft}^{BS}} 0.20.$$

[5]Lev and Gu (2016) write,

> *Revolutionary changes, shifting economies and business enterprises from the industrial to the information age, started to profoundly affect the business models, operations, and values of companies in the 1980s, yet, amazingly, triggered no change in accounting. Entire industries, which are largely intangible (conceptual industries, as Alan Greenspan called them), including software, biotech, and internet services, came into being during the 1980s and 1990s. And for all other businesses, the major value drivers shifted from property, plant, machinery, and inventories, to patents, brands, information technology, and human resources. The latter set, all missing from companies balance sheets because accountants treat intangible investments like regular expenses (wages, or interest), thereby distorts both the balance sheet and income statement. The constant rise in the importance of intangibles in companies performance and value creation, yet suppressed by accounting and reporting practices, renders financial information increasingly irrelevant.*

[6]However, there are some exceptions. For example, the US GAAP treats the development of computer software differently from other R&D costs. Following the ASC 985 (formerly FAS 2), once a software developer has reached technological feasibility, the developer must capitalize and amortize all development costs until the product becomes available for general release to consumers.

which spent around $16 billion in research and development and $12 billion in sales and marketing in 2017 (see Figure IIa and Figure IIb).

Figure II: Intangible Investments by Google

Research and Development

The following table presents our R&D expenses (in millions):

	Year Ended December 31,		
	2015	2016	2017
Research and development expenses	$ 12,282	$ 13,948	$ 16,625
Research and development expenses as a percentage of revenues	16.4%	15.5%	15.0%

R&D expenses consist primarily of:

- Compensation expenses, including SBC, and facilities-related costs for employees responsible for R&D of our existing and new products and services; and
- Depreciation and equipment-related expenses.

(a) Research and Development Expenses

Sales and Marketing

The following table presents our sales and marketing expenses (in millions):

	Year Ended December 31,		
	2015	2016	2017
Sales and marketing expenses	$ 9,047	$ 10,485	$ 12,893
Sales and marketing expenses as a percentage of revenues	12.1%	11.6%	11.6%

Sales and marketing expenses consist primarily of:

- Advertising and promotional expenditures related to our products and services; and
- Compensation expenses, including SBC, and facilities-related costs for employees engaged in sales and marketing, sales support, and certain customer service functions.

(b) Marketing Expenses

Overall, these figures prove that a lot of intangible capital investment that is simply expensed by firms, in accordance with the US GAAP, does not show up as capital on the balance sheet. To overcome this limitation in the accounting standards, we capitalize knowledge capital, as explained in the main text.

Externally acquired intangible capital can be capitalized on firms' balance sheets at the fair value according to the US GAAP under guidelines provided from ASC 350 (formerly FAS 142) and shows up in Compustat in the variable `INTAN`. According to Ewens et al. (2019), firms and accountants follow the guidelines provided in ASC 820 (formerly FAS 157) to mark externally acquired intangible capital on the balance sheet at its fair value at the time of the acquisition. Firms can choose among different methods to compute the fair value according to the US GAAP, and firms' choices must be disclosed in the appraisal notes for intangibles in the buyer's financial statements. Firms have three options to appraise the value of intangible assets: (i) estimating the replacement cost of the asset, (ii) comparing the asset to a similar asset whose price trades on the open market, or (iii) using the Discounted Cash Flow model, where earnings or cash flows are discounted by an appropriate discount rate. In particular, acquired intangible assets can be individually capitalized with the methodologies reported above if and only if they are identifiable, as documented in the ASC 805 notes. An intangible asset is identifiable if it meets either (i) the separability criterion, meaning it can be separated from the entity and sold, or (ii) the contractual-legal criterion, meaning that the control of the future economic benefits arising from the intangible asset is warranted by contractual or legal rights. In other words, IIA prices reflect fair or public value rather than value specific to the post-acquisition firm. Some examples of these identifiable intangible assets include brand names, customer lists, trademarks, Internet domain names, royalty agreements, patented technologies, and trade secrets. Other intangibles with a non-zero value, such as corporate culture, advertising effectiveness, and management quality, that fail to meet these criteria for identification are captured as goodwill on the buyer's balance sheet (`GDWL` in Compustat).

Figure III: Coca-Cola's Externally Purchased Intangibles

Coca-Cola Co.

Balance sheet: goodwill and intangible assets

US$ in millions

	Dec 31, 2019	Dec 31, 2018	Dec 31, 2017	Dec 31, 2016	Dec 31, 2015
Trademarks	9,266	6,682	6,729	6,097	5,989
Bottlers' franchise rights	109	51	138	3,676	6,000
Goodwill	16,764	10,263	9,401	10,629	11,289
Other	110	106	106	128	164
Indefinite-lived intangible assets	**26,249**	**17,102**	**16,374**	**20,530**	**23,442**
Customer relationships	344	185	205	392	493
Bottlers' franchise rights	341	30	213	487	604
Trademarks	177	186	182	228	211
Other	55	88	94	179	97
Definite-lived intangible assets, gross carrying amount	**917**	**489**	**694**	**1,286**	**1,405**
Accumulated amortization	(400)	(321)	(432)	(688)	(715)
Definite-lived intangible assets, net	**517**	**168**	**262**	**598**	**690**
Intangible assets	**26,766**	**17,270**	**16,636**	**21,128**	**24,132**

Based on:10-K (filing date: 2020-02-24),10-K (filing date: 2019-02-21),10-K (filing date: 2018-02-23),10-K (filing date: 2017-02-24),10-K (filing date: 2016-02-25).

We give an example of Coca-Cola's externally purchased intangibles in Figure III. Coca-Cola writes in their yearly report:

> *We classify intangible assets into three categories: (1) Intangible assets with definite lives subject to amortization, (2) intangible assets with indefinite lives not subject to amortization and (3) goodwill.*

The goodwill and intangible assets with indefinite lives are subject to an impairment test every period, and their values are increased or decreased accordingly. As one can see, the balance sheet intangibles are the sum of heterogeneous assets, such as trademarks, franchise rights, and customer relationships, among others.

Internally generated intangible capital: Potential issues. The fact that a sizeable fraction of intangible capital is internally produced and cannot be capitalized on firms' balance sheets potentially implies that some possible concerns related to the double-counting of some intangible assets. For example, when firm 1 produces its own intangible capital, it will expense it in its income statement at production cost x. If this intangible capital is then sold to firm 2, this sale will not show up in the income statement of firm 1 as a negative cost (or a negative investment). Firm 2 will, however, show this new intangible capital on its balance sheet at fair value y because it has been externally acquired. In this example, even though the overall amount of intangible capital has not changed in the economy—just a transaction has taken place—we would potentially observe an increase in the overall stock of intangible capital from x to $x + y$.

Although this is a concern in theory, as a practical matter, we are confident that this situation is rare and hence of little quantitative relevance. First, we know that intangible capital is often acquired through the acquisition of an entire firm.[7] Hence, as the target firm is acquired, it exits the sample, and its intangible assets leave the sample as well, whereas now the acquiring firm will show an increase in its intangible capital on the balance sheet. Second, we also know that a lot of intangible capital is acquired as final goods from other firms (consider, for instance, software producers and advertisement/marketing companies), and in this case as well, there is no double-counting as this

[7] Peters and Taylor (2017) and Ewens et al. (2019).

is final production and not internal production for firms' own use. Third, as we showed in the previous section, internally produced intangible capital is a declining feature of our empirical measure, suggesting that this concern should be minor and declining over time. Therefore, we conclude that this issue is not quantitatively appealing.

Externally acquired intangible capital: Potential issues. Externally purchased intangible capital is almost often acquired through acquisitions of entire firms, and this greatly influences the way it is capitalized on the firms' balance sheets. For example, imagine firm x buys firm y at cost p^y. At the moment of the acquisition, firm x has to place the acquired assets on its balance sheet. Normally, the procedure is: (i) tangible assets are identified and capitalized at the fair value p^T, (ii) identifiable intangible assets are capitalized at the fair value p^I, and (iii) the residual value is attributed to unidentifiable intangible assets (synergies, organizational culture, etc.) and capitalized into goodwill. Therefore, in the data, we have $\texttt{GDWL} = p^y - p^T - p^I$.

If a researcher thinks that firms acquire other firms to exercise future market power (and so firms are willing to pay high prices for them), the concern can arise that these unidentifiable intangible assets are just the discounted expected sum of the value of future market power, and therefore the value of balance sheet intangibles goes up by more than its quantity. One way to address this concern is to use proper deflators, that is, to deflate intangible capital with the IPP deflator.[8] However, this only takes care of aggregate common trends and cannot account for the heterogeneity in firm-level input prices, and unfortunately, more disaggregated investment deflators do not exist. We wish to emphasize that the inability to obtain firm-level investment deflators equally affects the measurement of tangible and intangible capital. Additionally, as a more appealing way to address these concerns, we remove goodwill from the total balance sheet of intangible capital as almost all of the potential rise in prices related to unidentifiable assets will be captured exactly by a rise in goodwill. However, we want to acknowledge that Ewens et al. (2019)—using more detailed data than we have—have shown that at least 38% of firms' goodwill is indeed true intangible capital. Therefore, we see this solution as a necessary but imperfect solution.

Accounting standards for software: A special case. The accounting standards for expenditures in internal software development or external purchases are different from those of other intangible assets. In particular, the FASB ASC subtopic 350-40 provides guidelines for the accounting of the costs for computer software developed or obtained for internal use and of the hosting arrangement obtained for internal use. The standards state that costs incurred during the development stage may be capitalized. Capitalization of the costs should cease in the post-implementation stage. The FASB ASC subtopic 985-20 provides guidelines for the accounting of the costs incurred for software meant to be sold, leased or marketed. The standards state that costs incurred subsequent to the establishment of technological feasibility may be capitalized. Capitalization of the costs should cease when the software is available for general release to customers.

To illustrate this, we provide an example of Athena Health Inc. software investments (see Figure IV). The company has capitalized software development costs of $113.9 million in 2017 and reports, external software acquisitions of $53.8 million in 2017.

Software used in research and development is subject to the subtopic 730-10. In general, software that is purchased from others and used for research and development activities and that has alternative futures/uses should be capitalized and amortized as an intangible asset. However, the cost for software purchased from others for a particular research and development project and that has

[8]This is standard practice in empirical work based on firm-level data.

Figure IV: Software Capitalization of Athena Health

6. CAPITALIZED SOFTWARE COSTS

Capitalized software consisted of the following:

	As of December 31,			
	2017		2016	
Capitalized internal-use software development costs	$	113.9	$	122.7
Acquired third-party software licenses for internal use		53.8		47.5
Total gross capitalized software for internal-use		167.7		170.2
Accumulated amortization		(74.8)		(82.9)
Capitalized internal-use software in process		46.8		38.5
Total capitalized software costs	$	139.7	$	125.8

Capitalized software amortization expense totaled $71.3 million, $73.5 million, and $53.4 million for the years ended December 31, 2017, 2016, and 2015, respectively. Future amortization expense for all capitalized software placed in service as of December 31, 2017 is estimated to be:

Years ending December 31,	Amount	
2018	$	50.5
2019		25.4
2020		10.7
2021		5.5
2022		0.8

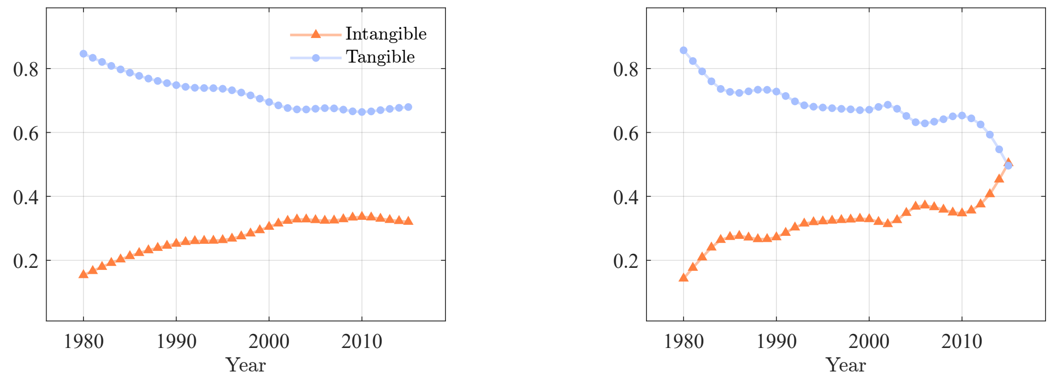
no alternative uses and therefore no separate economic value is considered a research and development cost and has to be expensed at the time it is incurred. In any case, we would capture most of the intangible capital related to software in our measure through balance sheet intangible capital or through capitalized knowledge capital.

I.I.IV Additional Validations for Firm-Level Intangible Capital

Here we compare some additional trends related to intangible capital investment, between aggregate data from the BEA and our measure from Compustat. Figure V compares the share of tangible capital investment into total investment and the share of intangible capital investment into total investment in both the BEA data and the Compustat data between 1980 and 2015. We can see that both data sources tell a similar story: in 1980, a substantial fraction of the investment was in tangible capital, whereas by 2015, tangible investment is roughly 70% of total investment in the BEA and 50% in Compustat. The two data sources tell similar stories, but they also show some discrepancies. In Compustat, the decline in the share of tangible capital investment of total investment is more pronounced; this could be due to, for instance, (i) the undercapitalization of true IPP capital in BEA or (ii) the selection of intangible-intensive firms in Compustat.

Figure VI shows the evolution of the different components of intangible capital investment in both the BEA and Compustat for the period 1980-2015. Again, the two data sources show a similar tendency: in 1980, most of the intangible capital investment was investment in research and development, whereas by 2015, investment in research and development accounts for less than 50% of total intangible capital investment. Finally, in Figure VII we compare the evolution of the intangible capital investment share across different sectors for both BEA data and Compustat data for the period 1998-2015. The sector-level intangible capital investment shares emerging from the Compustat data show trends similar to the ones computed with the BEA data. However, we see some differences in the level within some sectors. It is difficult to know what the sources of these discrepancies are; overall, we conclude that our firm-level measure of intangible capital does a reasonably good job of capturing the tendencies that are present in the aggregate data.

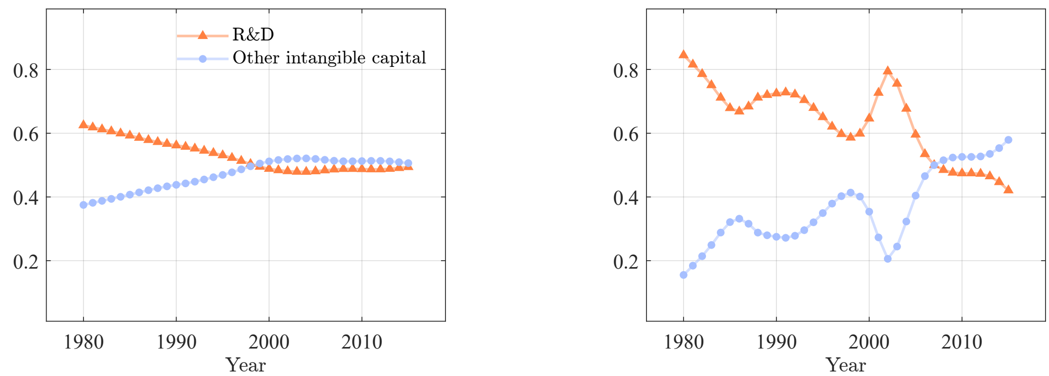
Figure V: Investment Components Share



(a) BEA (b) Compustat

Note. The figures show the evolution of the share of tangible capital investment and of intangible capital investment over total investment in both BEA data and Compustat data for the period 1980-2015. The data are detrended with an HP filter with $\lambda = 6.25$.

Figure VI: Intangible Capital Components Share



(a) BEA (b) Compustat

Note. The figures show the evolution of the share of knowledge capital investment (R&D) and other intangible capital investment (intangible capital investment different from R&D) over total intangible capital investment in both BEA data and Compustat data for the period 1980-2015. The data are detrended with an HP filter with $\lambda = 6.25$.

I.II Production Function Estimation

To estimate the firm-level production function, we follow De Loecker et al. (2020) and use two main approaches: (i) the control function approach and (ii) the cost shares approach. Both of these approaches are popular methods used to estimate firm-level production functions. Here we review the two methodologies, emphasizing their virtues and limitations.

I.II.I Ackerberg-Caves-Frazer

The control function approach has been pioneered by Olley and Pakes (1992) and developed further by Levinsohn and Petrin (2003) and Ackerberg et al. (2015). The main insight from this literature is that firm-level unobservable productivity can be proxied by some variable expenditure.

To overcome some of the criticism emphasized in Gandhi et al. (2020), we work with a structural value-added specification, as in Ackerberg et al. (2015) and De Loecker and Scott (2016), given by

$$\mathcal{Q}_{ft} = \min\left\{K^{\alpha}_{T,ft} K^{\nu}_{I,ft} L^{1-\alpha-\nu}_{ft} \exp(\omega_{ft} + \varepsilon_{ft}),\, \beta M_{ft}\right\}, \tag{2}$$

Figure VII: Intangible Capital Components Share by Sector

(a) Manufacturing (b) Information (c) Wholesale and Retail

(d) Professionals (e) Healthcare (f) Others

Note. The figures show the evolution of the intangible capital investment share across different sectors of the US economy for both BEA-KLEMS data and Compustat data between 1998 and 2015. The data are detrended with an HP filter with $\lambda = 6.25$.

where $\mathcal{Q}_{ft}$ is output, $K_{T,ft}$ is tangible capital, $K_{I,ft}$ is intangible capital, L_{ft} is labor, ω_{ft} is log productivity, ε_{ft} is the error term, and M_{ft} is material. This structural value-added production function yields the following first-order condition:

$$\mathcal{Q}_{ft} = K_{T,ft}^{\alpha} K_{I,ft}^{\nu} L_{ft}^{1-\alpha-\nu} \exp(\omega_{ft} + \varepsilon_{ft}), \tag{3}$$

justifying the regression of $\mathcal{Q}_{ft}$ on tangible capital, intangible capital, and labor while ignoring materials. A caveat is that, in theory, equation (3) may not be satisfied in certain situations. If both types of capital and labor are quasi-fixed and materials are a flexible input, then when output prices are sufficiently low relative to the price of materials, it will be better to set $M_{ft} = 0$ and not produce at all. However, given that our data only include actively producing firms, we assume that equation (3) always holds.[9] Therefore, under the specification in equation (2), the estimation of the firm-level production function reduces to

$$q_{ft} = \alpha k_{T,ft} + \nu k_{I,ft} + (1 - \alpha - \nu)\ell_{ft} + \omega_{ft} + \varepsilon_{ft}, \tag{4}$$

where $q_{ft} = \log(\mathcal{Q}_{ft})$, $k_{T,ft} = \log(K_{T,ft})$, $k_{I,ft} = \log(K_{I,ft})$, and $\ell_{ft} = \log(L_{ft})$. As usual, the main identification challenge to the production function estimation is the simultaneity bias induced by the unobserved time-varying firm-level productivity, ω_{ft}. We follow the control function literature, in particular, Ackerberg et al. (2015) and De Loecker et al. (2020), to estimate the production function in (4) using a two-step approach based on the use of a control function for the productivity process. The identification relies on the observation that a firm's tangible capital investment demand

[9] For a more detailed discussion on this issue, see Ackerberg et al. (2015).

is given by a policy function of the form $x_{T,ft} = x_T(k_{T,ft}, k_{I,ft}, \omega_{ft})$. Then, provided that the policy function is invertible, the productivity process can be proxied by a control function given by $\omega_{ft} = \omega(k_{T,ft}, k_{I,ft}, \omega_{ft})$ where $\omega(\cdot) = x_T^{-1}(\cdot)$.[10]

Therefore, in the first stage of this estimation procedure, we can clean the firm-level output value from measurement errors and unanticipated productivity shocks, regressing output on a polynomial of tangible capital, intangible capital, and labor, given by

$$q_{ft} = \mathcal{P}_t(k_{T,ft}, k_{I,ft}, \ell_{ft}) + \varepsilon_{ft}. \tag{5}$$

Then, in the second stage, using the estimate $\widehat{\mathcal{P}}_t$ from the previous stage, we can construct a measure of productivity that does not depend on the measurement error ε_{ft}, given by

$$\omega_{ft}(\alpha, \nu) = \widehat{\mathcal{P}}_t(k_{T,ft}, k_{I,ft}, \ell_{ft}) - \alpha k_{T,ft} - \nu k_{I,ft} - (1 - \alpha - \nu)\ell_{ft}. \tag{6}$$

Finally, taking advantage of the assumption that productivity follows an AR(1) process, it is possible to construct a measure of productivity innovations, given by

$$\xi(\alpha, \nu, \rho) = \omega_{ft}(\alpha, \nu) - \rho\omega_{ft-1}(\alpha, \nu). \tag{7}$$

Therefore, using the productivity innovations, we can construct a set of moment conditions to estimate the parameters of the production function, given by

$$\mathbb{E}(\xi(\alpha, \nu, \rho) \times \mathbf{z}_{ft}) = \mathbf{0}_{Z\times 1}, \tag{8}$$

where $Z \geq 3$ and, under the assumption that firms react to unanticipated productivity shocks contemporaneously and that capital is predetermined, the set of admissible instruments is $\mathbf{z}_{ft} \in \{\ell_{ft}, k_{T,ft}, k_{I,ft}, \ell_{it-1}, k_{T,ft-1}, k_{I,ft-1}, \dots\}$.

I.II.II Cost Shares

The cost shares approach has been prominently adopted in Foster et al. (2008) and exploits the first-order conditions of the firm. To make fruitful use of the first-order conditions of the firm, two assumptions are needed, namely: (i) constant returns to scale in production and (ii) all inputs are variable. Under these assumptions, the output elasticities can be calculated from cost shares. The cost shares of both inputs are defined as

$$\alpha = \text{med}\left\{\frac{r_t^T k_{T,ft}}{w_{ft}\ell_{ft} + r_t^T k_{T,ft} + r_t^I k_{I,ft}}\right\} \quad \text{and} \quad \nu = \text{med}\left\{\frac{r_t^I k_{I,ft}}{w_{ft}\ell_{ft} + r_t^T k_{T,ft} + r_t^I k_{I,ft}}\right\}, \tag{9}$$

where $w_{ft}\ell_{ft}$ is the wage bill, $r_t^T k_{T,ft}$ is the rental cost of tangible capital, and $r_t^I k_{I,ft}$ is the rental cost of intangible capital. Therefore, an extra requirement to apply this method is the possibility of calculating the return on both types of capital, r_t^T and r_t^I.

[10] The assumptions needed to ensure the invertibility of the policy functions associated with a wide class of production functions have been discussed extensively by Pakes (1991), Olley and Pakes (1992), Levinsohn and Petrin (2003), and Ackerberg et al. (2015).

I.III Robustness Production Function Estimation

In this subsection of the appendix we explain the alternative specifications that we use to test the robustness of IBTC. Results are presented in the main text.

I.III.I Unconstrained Returns to Scale

To test the robustness of our results to a more flexible specification of returns to scale we estimate with the ACF approach the following production function:

$$q_{ft} = \alpha k_{T,ft} + \nu k_{I,ft} + \beta \ell_{ft} + \omega_{ft} + \varepsilon_{ft}, \tag{10}$$

where the only difference compared with equation (4) is that now returns to scale are unconstrained. Therefore, with this alternative specification, the set of moment conditions becomes

$$\mathbb{E}(\xi(\alpha, \nu, \beta, \rho) \times \mathbf{z}_{ft}) = \mathbf{0}_{Z\times 1}, \tag{11}$$

where $Z \geq 4$.

I.III.II Sector-Level Production Technology

One restrictive assumption of our benchmark specification is that the production technology is the same across all sectors. We relax this assumption by allowing the production technology to be sector specific. Effectively, this means that we estimate the following production function:

$$q_{ft} = \alpha_s k_{T,ft} + \nu_s k_{I,ft} + (1 - \alpha_s - \nu_s)\ell_{ft} + \omega_{ft} + \varepsilon_{ft}, \tag{12}$$

which is identical to the benchmark one, except that now output elasticity is sector specific. Finally, with this specification, the average output elasticities will be computed using a sales-weighted average.

I.III.III Translog Production Function

We also test the robustness of our results to a more flexible production function: the translog production. This production function approximates a CES production function up to a second order. We choose a specification with constant returns to scale, given by

$$\begin{aligned} q_{ft} = {} & \alpha k_{T,ft} + \nu k_{I,ft} + (1-\alpha-\nu)\ell_{ft} \\ & - \beta k_{T,ft} k_{I,ft} - \beta k_{T,ft}\ell_{ft} - \beta k_{I,ft}\ell_{ft} + \beta k^2_{T,ft} + \beta k^2_{I,ft} + \beta \ell^2_{ft} + \omega_{ft} + \varepsilon_{ft}. \end{aligned} \tag{13}$$

Therefore, with this alternative specification, the set of moment conditions becomes

$$\mathbb{E}(\xi(\alpha, \nu, \beta, \rho) \times \mathbf{z}_{ft}) = \mathbf{0}_{Z\times 1}, \tag{14}$$

where $Z \geq 4$. Finally, the endogenous output elasticities will be given by

$$\theta^T = \mathrm{med}\big(\alpha - \beta k_{I,ft} - \beta \ell_{ft} + 2\beta k_{T,ft}\big), \tag{15}$$

$$\theta^I = \mathrm{med}\big(\nu - \beta k_{T,ft} - \beta \ell_{ft} + 2\beta k_{I,ft}\big), \tag{16}$$

$$\theta^\ell = \mathrm{med}\big(1 - \alpha - \nu - \beta k_{T,ft} - \beta k_{T,ft} + 2\beta \ell_{ft}\big). \tag{17}$$

I.III.IV Controlling for Output and Input Price Variation

It is well known that most of the time, standard production data, such as Compustat, record revenues and expenditures rather than physical production and input used. In the presence of product differentiation (be it through physical attributes or location), an additional source of endogeneity presents itself through unobserved output and input prices.[11] This implies that, when bringing the model to the data, the structural value-added production function takes the following form:

$$q_{ft} + p_{ft} = \alpha(k_{T,ft} + p_t^T) + \nu(k_{I,ft} + p_t^I) + (1-\alpha-\nu)(\ell_{ft} + p_{ft}^{\ell}) + \omega_{ft} + \varepsilon_{ft}, \tag{18}$$

where p_{ft} is the output price, p_t^T is the common user cost of tangible capital, p_t^I is the common user cost of intangible capital, and p_{ft}^{ℓ} is the price of labor. This empirical specification produces the following structural error term:

$$\omega_{ft} + p_{ft} - \alpha p_t^T - \nu p_t^I - (1-\alpha-\nu)p_{ft}^{\ell}. \tag{19}$$

We follow De Loecker et al. (2016) and let the wedge between the output and input price (scaled by the output elasticity) be a function of the demand shifters and the productivity difference.[12] The inclusion in the control function of demand shifters $\mathbf{d}_{ft}$, constructed using measures of market shares as in De Loecker et al. (2020), should therefore capture the relevant output and input market forces that generate differences in the output and input price. As discussed in De Loecker et al. (2016), this is an exact control when output prices, conditional on productivity, reflect input price variation and when demand is of the (nested) logit form.

In this case, the first-stage estimation procedure to clean from measurement error is given by

$$q_{ft} = \mathcal{P}_t(k_{T,ft}, k_{I,ft}, \ell_{ft}) + \boldsymbol{\vartheta}'\mathbf{d}_{ft} + \varepsilon_{ft}, \tag{20}$$

where $\mathcal{P}(\cdot)$ is a polynomial taking as inputs the firm's state variables and the control function, and $\mathbf{d}_{ft}$ is a vector of firm-level sales shares controlling for the pass-through of input price to output price variation. Under this alternative specification, in the second stage, using the estimates for $\widehat{\mathcal{P}}$ and $\widehat{\boldsymbol{\vartheta}}'$, we can construct a measure of productivity that does not depend on measurement error and unobservable input prices, given by

$$\omega_{ft} = \widehat{q}_{ft} - \alpha k_{T,ft} - \nu k_{I,ft} - (1-\alpha-\nu)\ell_{ft} - \widehat{\boldsymbol{\vartheta}}'\mathbf{d}_{ft}, \tag{21}$$

where $\widehat{q}_{ft} = \widehat{\mathcal{P}}(k_{T,ft}, k_{I,ft}, \ell_{ft}) + \widehat{\boldsymbol{\vartheta}}'\mathbf{d}_{ft}$. Notice that equation (21) is identical to the main specification up to the estimate of $\widehat{\boldsymbol{\vartheta}}'\mathbf{d}_{ft}$.

I.IV Robustness Investment Rates

In this section of the appendix, we show the detailed moments of the investment rate distribution of intangible capital produced by the robustness exercises described in the main text. In particular, we look at the investment rate distribution of intangible capital across different sectors, periods, firms of different ages and sizes, and different types of intangible capital, as well as at the investment rate distribution of intangible capital computed with a different methodology.

[11] See De Loecker and Goldberg (2014) and De Loecker et al. (2016) for a recent treatment of these issues.

[12] De Loecker et al. (2020) note that not observing output prices has the perhaps unexpected benefit that output price variation absorbs input price variation, thus eliminating part of the variation in the error term.

I.IV.I Investment Rates across Sectors

Table II: Investment Rates Moments by Sector

Investment rates	MIN	CON	MAN	TCU	WHO	RET	SRV
Average	0.23	0.20	0.37	0.34	0.19	0.17	0.30
Positive fraction, $i > 1$	0.62	0.88	0.91	0.83	0.76	0.75	0.89
Negative fraction, $i < -1$	0.05	0.09	0.02	0.03	0.9	0.07	0.04
Inaction rate	0.33	0.03	0.07	0.14	0.15	0.18	0.07
Spike rate, $\|i\| > 20$	0.52	0.67	0.82	0.61	0.51	0.45	0.70
Positive spikes, $i > 20$	0.51	0.58	0.81	0.60	0.49	0.42	0.69
Negative spikes, $i < -20$	0.01	0.09	0.01	0.01	0.03	0.03	0.01
Standard Deviation	0.27	0.25	0.23	0.33	0.25	0.27	0.25
Serial correlation, $\mathrm{Corr}(i_t, i_{t-1})$	0.39	0.45	0.32	0.33	0.21	0.14	0.34

Note. This table shows the moments of the investment rate distribution of intangible capital across different sectors. The statistics are computed for a balanced panel between 1980 and 1990. MIN is the mining sector. CON is the construction sector. MAN is the manufacturing sector. TCU is the transportation and public utilities sector. WHO is the wholesale sector. RET is the retail sector. SRV is the services sector.

Table II reports the investment rate distribution at the SIC1 level of disaggregation. In particular, we look at the mining, construction, manufacturing, transportation and public utilities, wholesale, retail, and services sectors. We find that in all sectors, although with some heterogeneity, intangible capital investment presents a higher positive spike rate and serial correlation relative to tangible capital investment. For example, even looking at the retail sector, which is the one presenting the lowest positive spike rate and serial correlation, we see that the rates are still 42% and 0.14, significantly higher than the rates associated with tangible capital. Notice that sectors or subsectors in which firms do not use intangible capital in production do not drive these patterns, as we exclude these firms when looking at the investment rate distribution. Hence, we are looking only at those firms within a sector that do use intangible capital.

I.IV.II Investment Rates across Time

Table III reports the investment rate distribution across different time periods. We find that the evolution of the investment distribution of intangible capital is remarkably stable over different time periods. This is particularly true for the positive spike rate (74% in the second decade and 69% in the last part of the sample) and for the serial correlation (0.25 in the second decade and 0.22 in the last part of the sample), which remain substantially higher than the ones associated with tangible capital investment for the entire period of our analysis.

I.IV.III Investment Rates across Firms of Different Age, Size, Leverage, and Liquidity Groups

Table IV reports the investment rate distribution across different groups of firms. In particular, we look at the behavior of the investment rate distribution of intangible capital between young and old firms, defined as firms below and above the median age, between small and large firms, defined as firms below and above median sales, between high and low leverage firms, defined as firms below and above median leverage, and between low and high liquidity firms, defined as firms below and above median liquidity. Leverage and liquidity are calculated as explained in Jeenas (2019). We find

Table III: Investment Rates Moments by Period

Investment rates	1991-2000	2001-2015
Average	0.34	0.32
Positive fraction, $i > 1$	0.89	0.90
Negative fraction, $i < -1$	0.03	0.06
Inaction rate	0.08	0.04
Spike rate, $\lvert i \rvert > 20$	0.75	0.72
Positive spikes, $i > 20$	0.74	0.69
Negative spikes, $i < -20$	0.01	0.03
Standard deviation	0.26	0.32
Serial correlation, $\text{Corr}(i_t, i_{t-1})$	0.25	0.22

Note. This table shows the moments of the investment rate distribution of intangible capital over time. The statistics are computed for a balanced panel of firms between 1980 and 1999 and between 2000 and 2015.

Table IV: Investment Rates Moments by Age, Size, Leverage, and Liquidity

Investment rates	Age		Size		Leverage		Liquidity	
	Young	Old	Small	Large	High	Low	Low	High
Average	0.37	0.33	0.35	0.35	0.31	0.37	0.33	0.37
Positive fraction, $i > 1$	0.85	0.89	0.84	0.91	0.85	0.92	0.88	0.91
Negative fraction, $i < -1$	0.02	0.03	0.02	0.02	0.03	0.01	0.02	0.01
Inaction rate	0.13	0.08	0.14	0.06	0.12	0.07	0.10	0.08
Spike rate, $\lvert i \rvert > 20$	0.72	0.79	0.70	0.82	0.70	0.84	0.78	0.79
Positive spikes, $i > 20$	0.71	0.77	0.69	0.81	0.69	0.84	0.77	0.79
Negative spikes, $i < -20$	0.01	0.02	0.01	0.01	0.02	0.00	0.01	0.01
Standard deviation	0.30	0.23	0.29	0.23	0.28	0.21	0.25	0.25
Serial correlation, $\text{Corr}(i_t, i_{t-1})$	0.27	0.32	0.29	0.34	0.37	0.38	0.35	0.31

Note. This table shows the moments of the investment rate distribution of intangible capital across different types of firms. The statistics are computed for a balanced panel of firms between 1980 and 1999. Young firms are firms with age below the median. Old firms are firms with age above the median. Small firms are firms with sales below the median. Large firms are firms with sales above the median. High leverage firms are firms with leverage above the median. Low leverage firms are firms with leverage below the median. Low liquidity firms are firms with liquidity below the median. High liquidity firms are firms with liquidity above the median.

that the moments of the investment rate distribution of intangible capital are stable across different groups of firms. Specifically, the positive spike rate is always higher than the one associated with tangible capital, ranging from a minimum of 69% for high leverage firms to a maximum of 84% for low leverage firms. Also, the serial correlation is higher than the one of tangible capital investment, going from a minimum of 0.27 for young firms to a maximum of 0.38 for low leverage firms. Overall, these findings suggest that the salient differences between the investment rate distribution of intangible and tangible capital are unlikely to be driven by the behavior of different groups of firms, such as young, small, high leverage, or low liquidity firms.

We notice that age, size, leverage, and liquidity have been interepreted by the literature as proxies for financial frictions (see Cloyne et al., 2023 for age; Gertler and Gilchrist, 1994, for size; Ottonello and Winberry, 2020, for leverage; and Jeenas, 2019, for liquidity). Hence, our findings also suggests that, while the role of intangible capital has been proven important in exacerbating financial existing financial frictions (see Falato et al., 2022, and Caggese and Pérez-Orive, 2022), the salient differences between the investment rate distribution of intangible and tangible capital do not seem to be driven by underlying differences in financial frictions.

I.IV.IV Investment Rates across Different Types of Intangible Capital

Table V: Investment Rates Moments by Type

Investment rates	BS	R&D
Average	0.23	0.35
Positive fraction, $i > 1$	0.88	0.83
Negative fraction, $i < -1$	0.11	0.00
Inaction rate	0.01	0.17
Spike rate, $\|i\| > 20$	0.53	0.77
Positive spikes, $i > 20$	0.48	0.77
Negative spikes, $i < -20$	0.05	0.00
Standard deviation	0.32	0.22
Serial correlation, Corr(i_t, i_{t-1})	0.19	0.47

Note. This table shows the moments of the investment rate distribution across different types of intangible capital. The statistics are computed for a balanced panel of firms between 1980 and 1999. BS is the balance sheet stock of capital. R&D is the stock of knowledge capital.

Table V reports the investment rate distribution of balance sheet intangible capital (k_{BS}) and knowledge capital ($k_{R\&D}$). Unsurprisingly, we find non-negligible differences between the two types of intangible capital. Part of these differences just highlight the heterogeneity present between different types of intangible capital, a pattern that we also observe in tangible capital investment structure and equipment have sizeable differences in their behavior; see the online appendix of Baley and Blanco (2021) for an example of the stark differences that different types of tangible capital, such as structure and equipment, can present. However, another part of these differences is just by construction, as knowledge capital is constructed by capitalizing an expenditure, which cannot be negative by definition; this observation explains why we do not observe negative investments in knowledge capital for example. Furthermore, we note that knowledge capital is what explains the inaction observed in intangible capital investments, hence our conservative choice to focus only on spike rates and serial correlation. However, despite these expected differences, both types of capital show patterns in their investment rate distribution that are in line with our benchmark findings. In particular, we find that both types of intangible capital present a higher positive spike rate compared to tangible capital, ranging from 48% of balance sheet capital to 77% of knowledge capital, and also a higher serial correlation compared to tangible capital, ranging from 0.19 for balance sheet capital to 0.47 for knowledge capital. Therefore, these findings suggest that, despite the expected heterogeneity across different types of intangible capital, the main patterns of our benchmark analysis are a persistent feature of any intangible investment stemming from its unique differences compared to tangible capital.

Table VI: Investment Rates Moments Alternative Calculations

Investment rates	Intangible	Tangible
Average	0.35	0.13
Positive fraction, $i > 1$	0.88	0.86
Negative fraction, $i < -1$	0.03	0.11
Inaction rate	0.08	0.03
Spike rate, $\lvert i \rvert > 20$	0.75	0.24
Positive spikes, $i > 20$	0.73	0.21
Negative spikes, $i < -20$	0.02	0.03
Standard deviation	0.30	0.21
Serial correlation, $\text{Corr}(i_t, i_{t-1})$	0.29	0.09

Note. This table shows the moments of the investment rate distribution of intangible and tangible capital. The statistics are computed for a balanced panel of firms between 1980 and 1990.

I.IV.V Investment Rates Calculated with Alternative Specification

Table VI reports the investment rate distribution of intangible and tangible capital when calculated with an alternative specification. In particular, in this specification, we calculate interest rates as described in the main text. We find that intangible capital still has a higher spike rate, 73% compared to 21% for tangible capital, and a higher serial correlation, 0.29 compared to 0.09 for tangible capital. These findings corroborate our benchmark results, suggesting that the way investment rates are calculated in the data does not play a role in the different behavior of the investment rate of these two types of capital.

I.IV.VI Investment Rates in the Presence of Measurement Error

Intangible capital measurement is not fully reported in the accounting standards, hence it may entail a non-trivial amount of measurement error in it. Here we test this hypothesis and its implications for the investment rate distribution. We allow for measurement error in the stock and re-evaluate the different moments of the distribution.

In particular, we assume that the intangible capital stock $k_{I,ft}$ is incorrectly measured, such that $k_{I,ft} = k^*_{I,ft} \exp(\omega_{ft})$ where $k^*_{I,ft}$ is the true intangible capital stock and $\exp(\omega_{ft})$ is the measurement error. Our specification follows Collard-Wexler and De Loecker (2021) and assumes that $\exp(\omega_{ft})$ is a classical measurement error. Hence, it is uncorrelated with true intangible capital stock but it is serially correlated over time. We assume that ω_{ft} follows a AR1 process given by

$$\omega_{ft} = \rho\omega_{ft-1} + \eta_{ft}, \quad \eta_{ft} \sim \mathcal{N}\left(0, \sigma^2_\eta\right); \tag{22}$$

where ρ is the persistence and σ^2_η is the variance of the i.i.d. component.

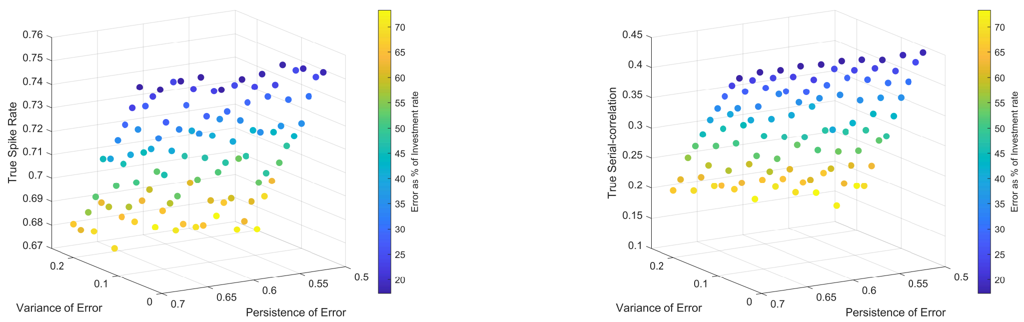
Under this measurement error specification, we can rewrite the true intangible capital investment rate as

$$\frac{x^*_{I,ft}}{k^*_{I,ft-1}} \approx \frac{x_{I,ft}}{k_{I,ft-1}} - (1-\rho)\omega_{ft-1} - \eta_{ft}. \tag{23}$$

As the parameters governing the measurement are unknown, we compute the spike rate and the correlation of the true investment under different values of persistence and the variance of the shock, simulating the measurement error process over many realizations, and plot them in Figure VIII.

Increasing variance and reducing persistence have the effect of amplifying the proportion of measurement error present in the observed capital stock, and consequently, in the recorded investment rate. We are interested in assessing how variations in the level of measurement error impact the actual, underlying investment rate. Our findings indicate that changes in measurement error do have an impact on the true value of the spike rate and the correlation. However, it's worth noting that this impact remains relatively modest, even when the measurement error constitutes a substantial portion of the observed investment rate in the data (computed as the absolute deviation averaged across firms), reaching up to approximately 70%. Notably, both the spike rate and the correlation, which are essential metrics for determining adjustment costs in the model, remain significantly higher compared to their counterparts in the tangible investment rate.

Figure VIII: True Spike Rate and Serial Correlation of Investment with Measurement Error



(a) True Spike Rate (b) True Serial Correlation

Note. Figure VIIIa and VIIIb show the true spike rate and true serial correlation of investment on the z-axis, respectively. The x-axis shows the value for persistence ρ and the y-axis shows the values for variance σ_η^2 of the measurement error. The legend (yellow to dark blue) reports the level of measurement error as the proportion of the observed investment rate in the data, where dark blue represents low levels of measurement error (approx. 20%) and yellow represents high level of measurement error (approx. 70%).

I.V Robustness for Marginal Revenue Product of Both Types of Capital

In this section, we explore the extent of the robustness of the results presented in the main text. In particular, we study how the presence of firm-level heterogeneous markups, a pervasive feature of the data (see De Loecker et al., 2020), can affect our measurement of the marginal revenue product of both types of capital.

In the presence of firm-level heterogeneous markups, but absent any adjustment friction, we obtain the following relation between the marginal revenue product of capital, markups, and the marginal cost of capital:

$$r + \delta_j \propto \frac{MRPK_j}{\mu}, \quad j \in \{T, I\}, \tag{24}$$

where r is the risk-free rate, δ_j is the capital specific depreciation rate, $MRPK_j$ is the marginal revenue product of both types of capital, and μ is the markups.[13] Equation (24) entails a logic similar to the one used in the main text, but now the relevant object of interest is not the $MRPK_j$ directly but the ratio $MRPK_j/\mu$. In the rest of the section, we will refer to the ratio $MRPK_j/\mu$ as the adjusted marginal revenue product of capital.

[13]This relation holds when firms rent capital competitively and in the absence of any friction to adjust capital, such as adjustment costs or financial frictions; see Hsieh and Klenow (2009) and Bau and Matray (2020).

I.V.I Responsiveness

To test the robustness of our second result in the main text, we notice that in the absence of any investment frictions such as adjustment costs, equation (24) holds, and hence the adjusted marginal revenue product of capital is proportional to $r + \delta_j$, which is uncorrelated with productivity innovations. The intuition is that the firms absorb productivity shocks by adjusting either their capital stock or their markup, such that equation (24) is satisfied. However, in the presence of adjustment costs that constrain the ability of the firm to adjust capital, equation (24) no longer holds with equality, and the adjusted marginal revenue product of capital becomes correlated with productivity shocks. Hence, if intangible capital is subject to higher adjustment costs compared to tangible capital, we should observe that the adjusted marginal revenue product of intangible capital is more correlated with productivity shocks relative to that of tangible capital.

Table VII: Heterogeneous Response of $MRPK_T/\mu$ and $MRPK_I/\mu$ to $TFPR$ Shocks

	Baseline Specification		Alternative Specification	
	(1)	(2)	(3)	(4)
Dependent Variable	$MRPK_{T,ft}/\mu_{ft}$	$MRPK_{I,ft}/\mu_{ft}$	$MRPK_{T,ft}/\mu_{ft}$	$MRPK_{I,ft}/\mu_{ft}$
ε_{ft}	0.84***	1.11***	0.81***	1.13***
	(0.01)	(0.01)	(0.00)	(0.01)
$k_{T,ft-1}$	-0.64***		-0.21***	
	(0.00)		(0.01)	
$k_{I,ft-1}$		-0.68***		-0.13***
		(0.00)		(0.00)
$TFPR_{ft-1}$	0.06***	0.03***	0.41***	0.80***
	(0.00)	(0.01)	(0.01)	(0.01)
ω_{ft}			0.57***	0.71***
			(0.00)	(0.00)
Time dummies	✓	✓	✓	✓
Firm dummies	✓	✓	✓	✓
Controls	✓	✓	✓	✓
Observations	88,964	88,964	80,485	80,485

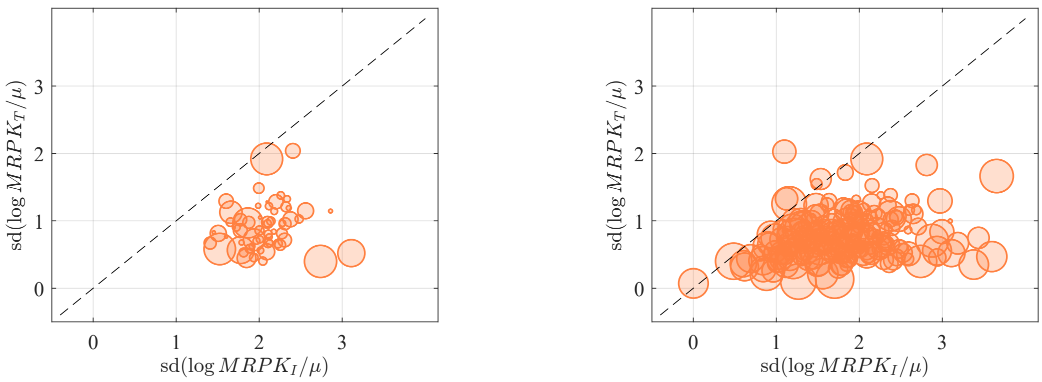
Notes. We report the coefficients from the regressions of marginal revenue product of tangible capital, $MRPK_{T,ft}$, and marginal revenue product of intangible capital, $MRPK_{I,ft}$, on revenue productivity shocks, ε_{ft}. The controls include sales, leverage, and liquidity. The baseline specification, which controls for classical (fixed and iid) measurement error, is shown in the main text. The alternative specification, which controls for serially correlated measurement error, is presented also inthe main text. Standard errors are in parentheses. *** p-value<0.01, ** p-value<0.05, * p-value<0.1.

To study the response of our adjusted marginal revenue product of both types of capital, we reestimated equations the main text with our alternative measures. Table VII presents the results. Columns 1-4 present the regression coefficients from the baseline specification with and without controls. Columns 5-6 present the regression coefficients from the alternative specification. We find that the adjusted marginal revenue product of both tangible and intangible capital responds positively to revenue productivity shocks, as γ_1 is significantly greater than zero in all specifications. Moreover, we see that the adjusted marginal revenue product of intangible capital is more reactive to revenue productivity shocks relative to the adjusted marginal revenue product of tangible capital. This result is consistent with the findings presented in the main text, suggesting that firm-level markups do not affect our main result. This finding corroborates the evidence that intangible capital is subject to higher adjustment frictions, such as adjustment costs, compared to tangible capital.

I.V.II Dispersion

To test the robustness of our first result in the main text, we notice that (24) implies that in the absence of any investment frictions, such as adjustment costs, the adjusted marginal revenue product of capital should equalize across firms and hence should not present any within-sector variation. However, if instead intangible capital investment is subject to higher adjustment costs compared to tangible capital investment, we should expect to observe a higher within-sector dispersion in the adjusted marginal revenue Product of intangible capital relative to tangible capital.

Figure IX: Sector-Level Dispersion in $MRPK_I$ and $MRPK_T$



(a) SIC2-Level (b) SIC3-Level

Note. The figures show the standard deviation of $MRPK_I/\mu$ (x-axis) and the standard deviation of $MRPK_T/\mu$ (y-axis). Standard deviations are calculated within sectors and averaged across the years. Marginal revenue products are constructed as described in the text. The dashed black line shows the 45-degree line. Figure IXa is constructed by calculating standard deviations at the SIC2 level; each circle represents a SIC2 sector, where the size of the circle is proportional to its size (sale weighted) in Compustat. Figure IXb is constructed by calculating standard deviations at the SIC3 level; each circle represents a SIC3 sector, where the size of the circle is proportional to its size (sale weighted) in Compustat.

Figure IX presents the results of our robustness exercise where markups have been calculated as in De Loecker et al. (2020). Both figures show that in the vast majority of sectors, if we consider both SIC2 and SIC3 levels of disaggregation, the adjusted marginal revenue product of intangible capital is more dispersed than that of tangible capital. Therefore, we conclude that dispersion in firm-level markups does not affect our results. This finding comes from the fact that firm-level markups influence both measures equally and hence do not play any major role in their relative dispersion.

I.VI Aggregate Trends

In this section, we present the evolution between 1980 and 2015 of the main trends of interest for the quantitative analysis. For the trends constructed with the Compustat data, we explain the measurement procedure; for the others, we just refer to main papers that document them. In particular, we look at: (i) the rise in concentration, (ii) the decline in the labor share, (iii) the rise in the intangible capital investment share, (iv) the decline in the tangible capital investment share, (v) the decline in the tangible capital investment rate, (vi) the rise in the profit rate, (vii) the rise in the average firm size, and (viii) the decline in the allocative efficiency of the economy, that is, the rise in the standard deviation of $TFPR$.

We measure concentration using the HHI, as in Grullon et al. (2019). In Compustat, the HHI of sector s is constructed as

$$HHI_{st} = \sum_f \left(\frac{\texttt{SALE}_{ft}}{\sum_f \texttt{SALE}_{ft}} \right). \tag{25}$$

Then, the aggregate concentration is simply the sales-weighted average of the sector-level concentrations.[14]

The firm-level profit rate, adjusted for intangible capital, is defined as

$$\pi_{ft} = \frac{\texttt{SALE}_{ft} - \texttt{COGS}_{ft} - (\texttt{XSGA}_{ft} - \texttt{XRD}_{ft}) - r_{T,t}k_{T,ft} - r_{I,t}k_{I,ft}}{\texttt{SALE}_{ft}}. \tag{26}$$

The construction of the user cost of both types of capital is described in Appendix I.I.II. We drop XRD from XSGA to avoid double-counting research and development costs as they are both part of our measured intangible capital costs and our selling, general, and administrative costs. The standard unadjusted profit rate is instead defined as

$$\pi_{ft} = \frac{\texttt{SALE}_{ft} - \texttt{COGS}_{ft} - \texttt{XSGA}_{ft} - r_{T,t}k_{T,ft}}{\texttt{SALE}_{ft}}. \tag{27}$$

To obtain the aggregate profit rate, we use a sales-weighted average of both measures of the firm-level profit rate.

Finally, to measure the allocative efficiency of the US economy, we measure the standard deviation of $TFPR$. We compute $TFPR$ as

$$TFPR_{ft} = \log \texttt{SALE}_{ft} - \alpha k_{T,ft} - \nu k_{I,ft} - (1 - \alpha - \nu) \log \texttt{EMP}_{ft}, \tag{28}$$

where α and ν are the estimates from the main text. Then, our measure of allocative efficiency is just the dispersion in $TFPR$ over the different years. Figure Xh shows the evolution of these trends between 1980 and 2015.

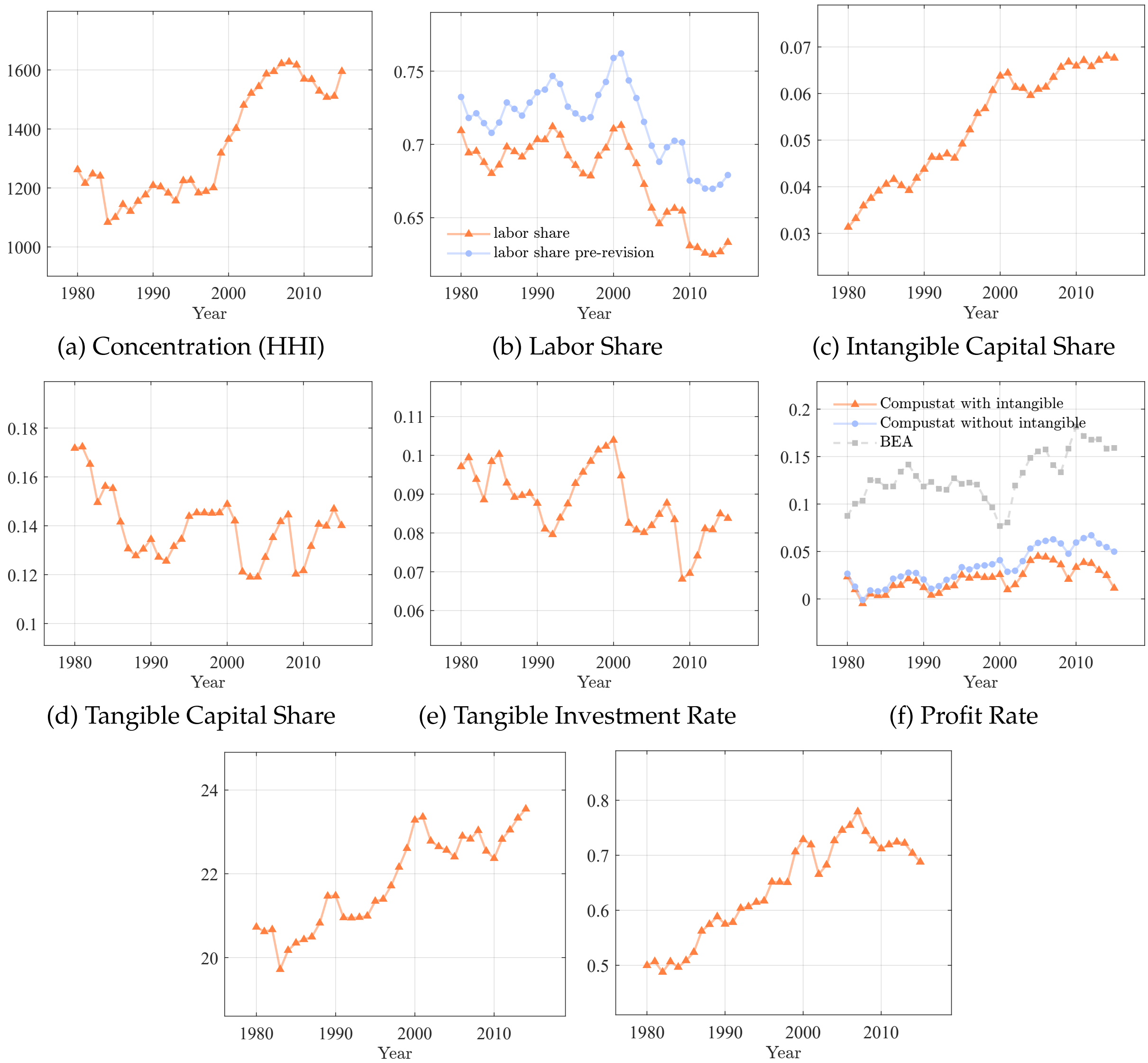
I.VII Patents and Intangible Investments: A Case Study

In this section, we focus on one particular type of intangible investment, patents. We use the patent applications and grant dataset from Autor et al. (2020) and merge with Compustat to get the yearly number of patent applications and grants for Compustat firms. Because of data limitations, we restrict our final sample to the period between 1977 and 2013. According to the dataset, 16% of the observations have at least one patent application. We assign zero to missing values of patents in the final dataset. Further, we build the stock of patents by employing the perpetual inventory method, as used in the main text. We employ depreciation rates of R&D investments. We also use alternative depreciation rates, and the main results remain qualitatively similar. We use the yearly number of patents to validate our baseline measure of firm-level intangibles. In Table VIII, we present a series of correlations between different measures of firm-level patents and different measures of firm-level intangible capital and investment. We find that all patent measures are robustly and significantly correlated with our measures of intangibles, supporting the meaningfulness of our measures.

Further, we compute the investment rate of patents (gross change in the measure of patents over the total stock of patents, as in the main empirical specification). Results are reported in Table IX. We find patterns that qualitatively resemble those of our benchmark measure of intangible capital investment. In particular, the average investment rate, the spike rate, and the serial correlation are in line with those of intangible capital investment and much higher relative to those of tangible capital investment. This finding corroborates the notion that these moments are a robust feature of intangible

[14] We follow Grullon et al. (2019) and use the two-digit NAICS level as the definition of the sector level.

Figure X: Aggregate Trends



(a) Concentration (HHI) (b) Labor Share (c) Intangible Capital Share

(d) Tangible Capital Share (e) Tangible Investment Rate (f) Profit Rate

(g) Avg. Firm Size (h) sd(log $TFPR$)

Note. Figure Xa replicates the evolution of the HHI in Compustat, as documented by Grullon et al. (2019). Figure Xb shows the evolution of the labor share, pre- and post-revision, in the corporate non-financial sector, as reported in Koh et al. (2020). Figure Xc shows the evolution of the intangible capital investment share in the corporate non-financial sector, as reported in Koh et al. (2020). Figure Xd shows the evolution of the tangible capital investment share in the corporate non-financial sector, as reported in Koh et al. (2020). Figure Xe shows the evolution of the tangible capital investment rate, as reported by Crouzet and Eberly (2019). Figure Xf shows the evolution of the profit rate, as reported in De Loecker et al. (2020) and the profit rate adjusted for intangible capital. Figure Xg shows the evolution of the average firm size measured in number of employees from BDS data. Figure Xh shows the evolution of the standard deviation of $TFPR$ in Compustat.

capital and do not depend on the measure at hand.

II Quantitative Appendix

II.I Alternative Modeling Assumptions

II.I.I Intangible Capital as a Demand Shifter

In this section, we show that introducing intangible capital as a demand shifter in the presence of a standard CES demand would lead to a model that is isomorphic to the one used in the empirical part

Table VIII: Correlation between Patents and Our Measure of Intangibles

Variables	(1) Intangible Investment	(2) Intangible Investment	(3) Intangible Capital	(4) Intangible Capital	(5) Intangible Investment	(6) Intangible Investment	(7) Intangible Capital	(8) Intangible Capital
Patent applications	2.63***				0.74***			
	(0.04)				(0.07)			
Patent grants		3.02***				1.58***		
		(0.04)				(0.07)		
Patent stock (applications)			3.47***				1.32***	
			(0.04)				(0.05)	
Patent stock (grants)				4.03***				2.81***
				(0.04)				(0.05)
Observations	52,352	52,352	52,352	52,352	52,190	52,190	52,190	52,190
R-squared	0.07	0.09	0.12	0.16	0.34	0.35	0.70	0.71
Firm dummies					✓	✓	✓	✓
Time dummies					✓	✓	✓	✓

Notes. We report the coefficients of correlation between our baseline measures of intangibles and different measures of firm-level patents. Standard errors are in parentheses. *** p-value<0.01, ** p-value<0.05, * p-value<0.1.

Table IX: Investment Rates Moments Patents

Investment rates	Patents
Average	0.28
Positive fraction, $i > 1$	0.58
Negative fraction, $i < -1$	0.00
Inaction rate	0.42
Spike rate, $\lvert i \rvert > 20$	0.49
Positive spikes, $i > 20$	0.49
Negative spikes, $i < -20$	0.00
Standard deviation	0.36
Serial correlation, $\mathrm{Corr}(i_t, i_{t-1})$	0.18

Note. This table shows the moments of the investment rate distribution of patents. The statistics are computed for a panel of firms between 1980 and 2013.

of the main text and in the quantitative theoretical part of the main text; see De Loecker (2011) for a similar empirical approach. We assume that the demand function faced by firm f is given by

$$q = p^{-\sigma} k_I^{\nu} C, \tag{29}$$

where k_I is the stock of intangible capital, p is the price charged by the firm, and C is aggregate consumption. Notice that this demand function can easily be microfounded through a standard CES structure, where intangible capital influences the value that the final consumer experiences from a given variety; see Sedláček and Sterk (2017) for a potential microfoundation of this demand structure. The production technology, in this case, is simply given by

$$q = e^z k_T^{\alpha} \ell^{1-\alpha}, \tag{30}$$

where k_T is tangible capital, ℓ is labor, and z is the idiosyncratic productivity. In this environment, the static profit maximization problem of the firm is

$$\begin{aligned} \pi &= \max_{p,\ell} pq - W\ell, \\ q &= p^{-\sigma} k_I^{\nu} C, \\ q &= e^z k_T^{\alpha} \ell^{1-\alpha}; \end{aligned} \tag{31}$$

which can alternatively be restated as

$$\pi = \max_{\ell} e^{\widehat{z}}\, k_T^{\widehat{\alpha}} k_I^{\widehat{\nu}} \ell^{\widehat{1-\alpha-\nu}} - W\ell, \tag{32}$$

where $e^{\widehat{z}} \equiv e^{z\frac{\sigma-1}{\sigma}} C^{\frac{1}{\sigma}}$, $\widehat{\alpha} \equiv \alpha(\sigma-1)/\sigma$, $\widehat{\nu} \equiv \nu/\sigma$, and $\widehat{1-\alpha-\nu} \equiv (1-\alpha)(\sigma-1)/\sigma$. Hence, the problem stated in equation (32) is isomorphic to the problem stated in the main text of the benchmark model and proposes an additional rationalisation of the empirical approach proposed the main text.

II.I.II Intangible Capital, Returns to Scale, and Market Power

In this section, we show that in the presence of a standard CES demand the presence of empirically plausible increasing returns to scale would lead to a model that is isomorphic to the one used in the quantitative theoretical part of the main text. We assume that the demand function faced by firm f is given by

$$q = p^{-\sigma} C, \tag{33}$$

where p is the price charged by the firm and C is aggregate consumption. Notice that this demand function can easily be microfounded through a standard CES structure. The production technology, in this case, is simply given by

$$q = e^z \left(k_T^{\alpha} k_I^{\nu} \ell^{1-\alpha}\right)^{\omega}, \tag{34}$$

where k_T is tangible capital, k_I is intangible capital, ℓ is labor, and z is the idiosyncratic productivity. In this environment, the static profit maximization problem of the firm is

$$\begin{aligned} \pi &= \max_{p,\ell} pq - W\ell, \\ q &= p^{-\sigma} C, \\ q &= e^z \left(k_T^{\alpha} k_I^{\nu} \ell^{1-\alpha}\right)^{\omega}; \end{aligned} \tag{35}$$

which can alternatively be restated as

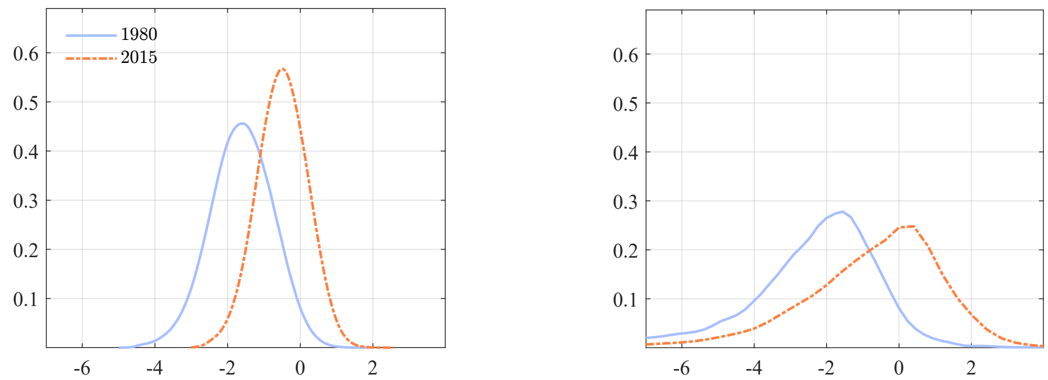
$$\pi = \max_{\ell} e^{\widehat{z}} \left(k_T^{\alpha} k_I^{\nu} \ell^{(1-\alpha-\nu)}\right)^{\widehat{\omega}} - W\ell, \tag{36}$$

where $e^{\widehat{z}} \equiv e^{z\frac{\sigma-1}{\sigma}} C^{\frac{1}{\sigma}}$ and $\widehat{\omega} = \omega(\sigma-1)/\sigma$ is the curvature of the revenue function. Hence, the problem stated in equation (36) is isomorphic to the problem stated in the main text of the benchmark model. This finding shows that calibrating a competitive economy with decreasing returns to scale (for example, 0.90) or a monopolistically competitive model with CES demand, mild increasing returns to scale (for example, 1.10), and empirically meaningful market power (for example, 1.22) is observationally equivalent.

II.II Additional Comparisons between Model and Data over Time

In this section, we document two additional implications of the model over time and compare them with the data. In particular, we look at the distribution of intangible intensity, defined as the ratio of intangible capital to the labor bill, and at the distribution of $TFPR$.

Figure XI: Intangible Intensity

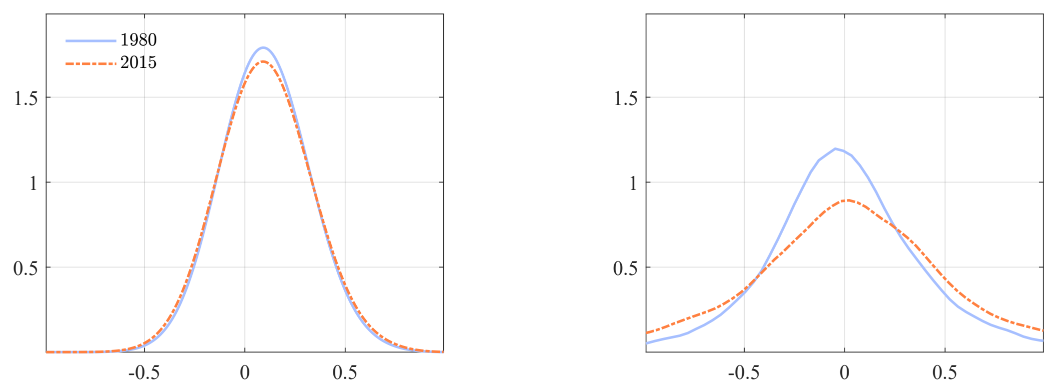


(a) Model (b) Data

Note. Figure XIa shows the distribution of log intangible intensity in be 1980 (solid light blue line) and 2015 (dashed orange line) from the model. Figure XIb shows the same distributions from the data.

Figure XI shows the evolution over time of the distribution of intangible intensity in both the model and the data. Overall, despite some qualitative differences, both the model and the data show a shift toward the right in the distribution of intangible intensities, highlighting the fact that firms on average are using more intangible capital relative to labor.

Figure XII: Total Factor Productivity Revenue



(a) Model (b) Data

Note. Figure XIIa shows the distribution of $TFPR$ in 1980 (solid light blue line) and 2015 (dashed orange line) from the model. Figure XIIb shows the same distributions from the data. All distributions are demeaned.

Figure XIIa shows the evolution, in both the model and the data, of the distribution of $TFPR$. In both the model and the data, the distribution of $TFPR$ is more dispersed in 2015, highlighting a decline in allocative efficiency. This, as emphasized in the main text, is a result of firms relying more on an input that is highly dispersed because of technological constraints, which hinders a fast reallocation of inputs toward high marginal product firms.

II.III Additional Robustness Quantitative Implications of IBTC

In this section, we report the calibrations of the robustness exercises performed in the main text. Table X reports the moments generated by the different calibrations together with the moments from the data. Table XI shows the parameters used in the different calibrations.

Table X: Alternative Calibrations: Moments

Target Moments	1980 Convex Costs Only	1980 Matching Inaction Rates	1980 Data	2015 Alternative Adj. Costs	2015 Data
Investment Rate Distributions					
Average investment rate x_T	0.18	0.18	0.11	0.15	0.11
Average investment rate x_I	0.39	0.38	0.34	0.38	0.32
corr $(x_{T,ft}, x_{T,ft-1})$	0.09	0.10	0.09	0.09	0.09
corr $(x_{I,ft}, x_{I,ft-1})$	0.31	0.31	0.31	0.23	0.22
Positive spike rate x_T	–	–	0.19	0.23	0.19
Positive spike rate x_I	–	–	0.76	0.55	0.75
Inaction rate x_T	–	0.03	0.03	–	–
Inaction rate x_I	–	0.11	0.10	–	–
Firm Dynamics					
Entry rate	0.11	0.10	0.13	–	–
Average firm size	21.0	20.7	20.5	–	–
Average entrant size	5.96	6.15	6.07	–	–
Wage	1.00	1.00	–	–	–

Table XI: Alternative Calibrations: Parameters

Fitted Parameters	Convex Costs Only	Matching Inaction Rates	Alternative Adj. Costs	Description
	Value			
Investment Adjustment Costs				
γ_T	0.038	0.039	0.045	Convex adjustment cost k_T
γ_I	0.680	0.700	0.370	Convex adjustment cost k_I
f_T	0	0.002	0.030	Fixed adjustment cost k_T
f_I	0	0.022	0.035	Fixed adjustment cost k_I
Firm Dynamics				
c_e	0.120	0.120	–	Entry cost
c_f	1.950	1.950	–	Operating cost
η	2.705	2.710	–	Scale parameter
m	0.024	0.025	–	Measure of potential entrants

We find that the different calibrations satisfactorily match the targeted moments. Moreover, we notice that all the calibrations infer that higher adjustment costs are associated with intangible capital investments relative to tangible capital ones. In particular, when using inaction rates, we find that the model infers that the fixed adjustment cost associated with intangible capital is higher than that associated with tangible capital, and this difference is starker than the one coming from the benchmark calibration. This finding leads us to conclude that targeting spike rates across different types of capital has to be interpreted as a conservative calibration choice. We conclude that the micro-level property that intangible capital investment is subject to higher adjustment costs compared to tangible capital investment is a robust property of this new capital that is robust to alternative calibration strategies.

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